

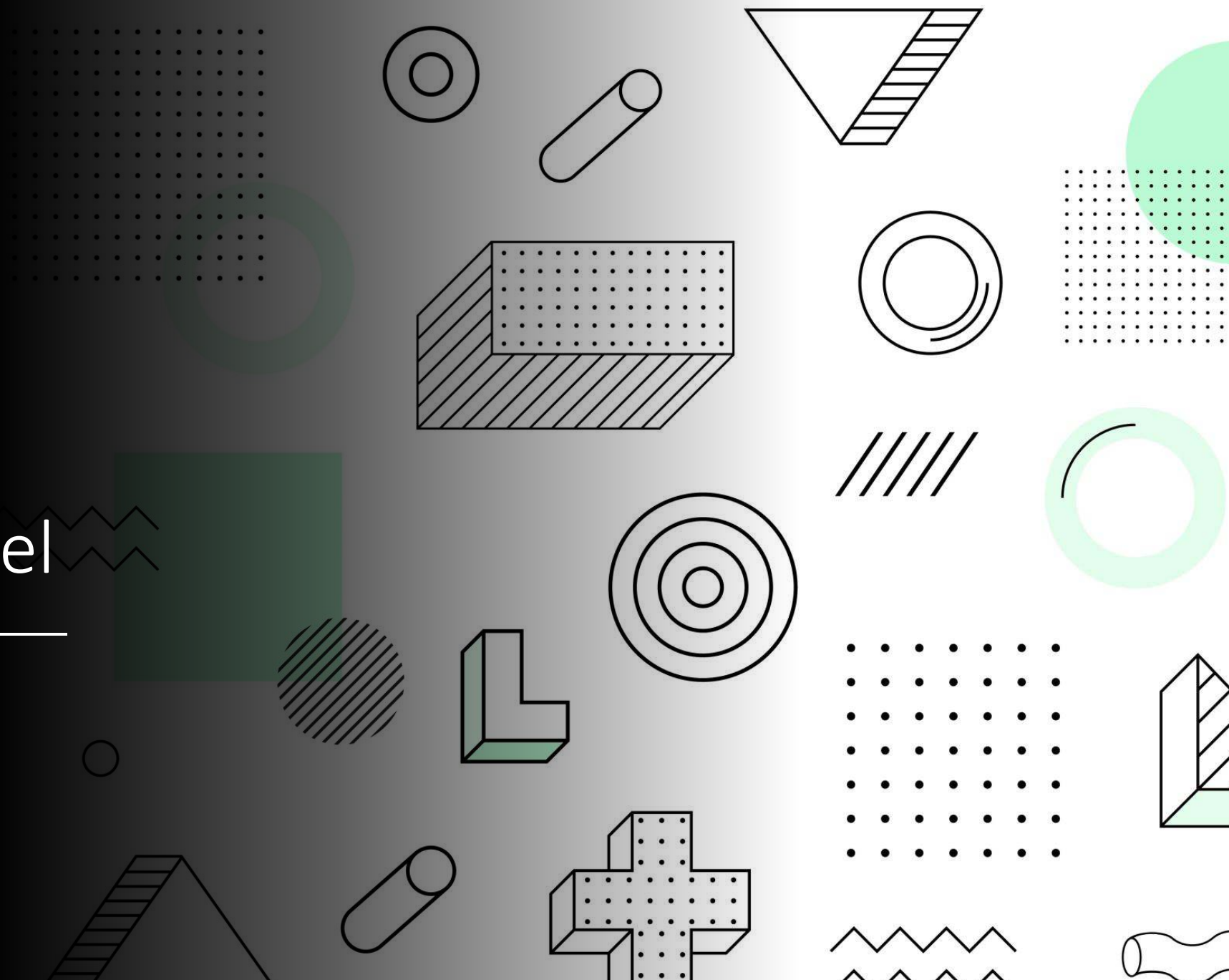


Reframing the teaching and learning of high school algebra with the 5E model

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SCI5233 Inventing and Reinventing Mathematics and Science Curriculum



Introduction

- Application of the three levels of inquiry (structured, guided and open) to mathematics is discussed generally in Wathall (2016)
- Application of inquiry-based learning to algebra instruction in a remedial algebra college course by Piercey (2017) resulted in those students performing as well on average as the students who were placed into the class above them
- Tuna and Kaçar (2013) found that applying the 5E model to the teaching of 10th grade trigonometry improved both performance in the unit summative and long-term retention of trigonometric knowledge
- Teaching the rules of algebra is seen as a dry, unengaging “necessary evil”, and no specific examples of the 5E model applied to algebra could be found in the literature

Phases of the 5E model ([BSCS Science Learning, 2020](#))



Engagement – activate prior knowledge and spark curiosity



Exploration – “play” with the topic in a way that leads to new understanding



Explanation – precisely define the new concept(s)



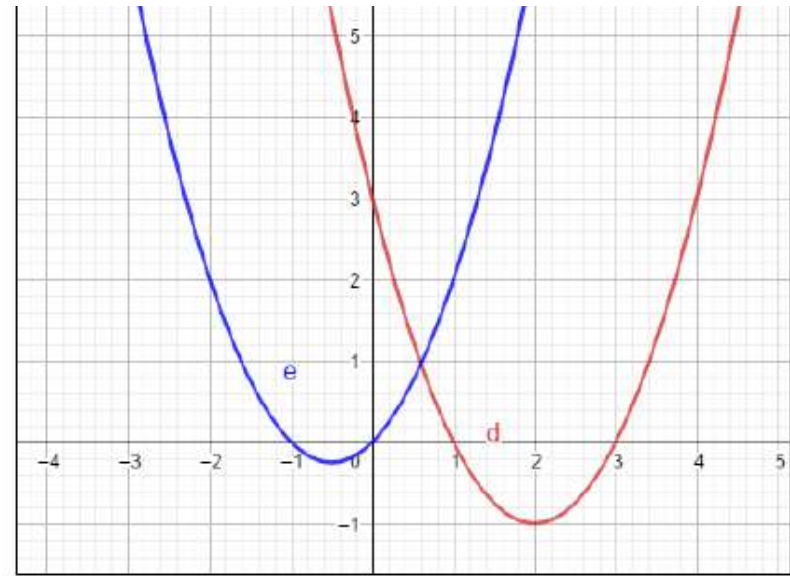
Elaboration – stretch, challenge, apply new knowledge



Evaluation – assessment (both self- and summative), reflection, updating the big picture

Engagement

- Students engage with the new topic via focused inquiry questions rather than teacher-led examples
- A blend of hand-written mathematics and use of software tools (Geogebra) allows students to make the connection between algebraic and graphical solutions from the outset



Questions for discussion

- Why does $y = (x + p)(x + q)$ have roots at $x = -p$ and $x = -q$?
- How can you find values of b , c , p and q so that the two graphs are the same?
- Are there any values of b and c where $y = x^2 + bx + c$ can't be written as $y = (x + p)(x + q)$?

What is the y -value when $x = -p$ or $x = -q$?

Try changing b and c and then predicting what p and q would need to be.

Is it possible for a quadratic graph not to have roots?

Figure 1. Example Geogebra activity designed for pre-calculus students ([Mathematics Education Innovation, 2018](#)).

Exploration

- Graspable Math allows students to intuitively discover how algebra works by playing around with it, while still ensuring that they play by the rules ([Graspable Math, 2019](#))
- Activities in which students try to manipulate an algebraic expression into a desired goal state lay the foundations for algebraic proof

11. Introduction to Factoring

This activity teaches students how to factor in GM

This level introduces you to some new GMA gestures, such as factoring a single term or a parentheses term. The focus of this level is factoring, and the objective is for you to transform the starting expression to match the goal.

Tasks

1. Multimedia

Apply the distributive property to factor an entire expression.
Drag the terms that share a common factor on top of each other to factor

2. Goal state

$3 \cdot 7 + 3 \cdot 4$
 $\hookrightarrow 3(7 + 4)$

4. Goal state

$10 - 25t$
 $\hookrightarrow 5(2 - 5t)$

5. Goal state

$2(12 + 6a)$
 $\hookrightarrow 2 \cdot 6(2 + a)$

7. Goal state

8. Goal state

Figure 2. Example goal state-style activity suitable for middle school students ([Graspable, Inc., 2020](#)). Note: access to this resource requires a subscription.

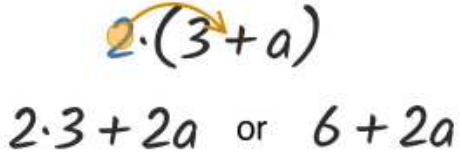
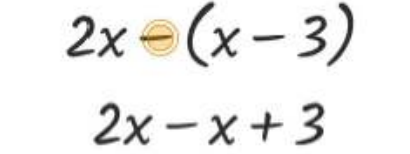
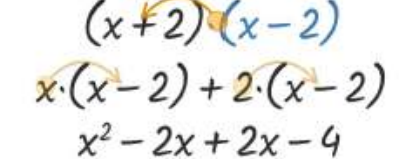
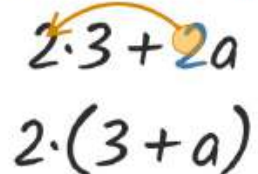
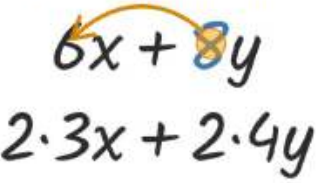
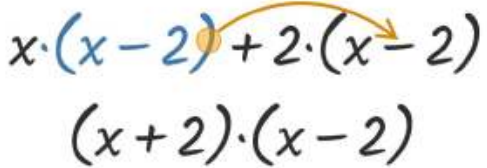
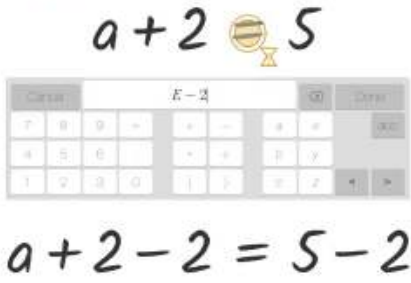
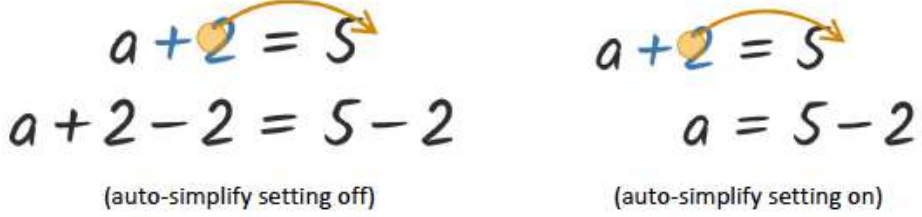
<p>Distribute a Term Drag a term into parentheses to distribute. Automatically simplifies in advanced settings</p> 	<p>Distribute a Negative Sign Tap on the negative sign to distribute.</p> 	<p>Distribute Multiple Terms Drag one sum in another to distribute. Double-tap "(" instead to skip line 2 below.</p> 
<p>Factor a Term Drag common terms onto each other to apply the distributive property.</p> 	<p>Finding Greatest Common Factors When numbers don't match, GM finds their greatest common factor.</p> 	<p>Factoring Multiple Terms You can factor groups of terms, too.</p> 
<p>Equations: Do the Same to Both Sides I Tap and hold the "=" and enter an operation to apply to both sides in the keypad.</p> 	<p>Equations: Do the Same to Both Sides II Drag a term to the other side of the equation to apply the inverse.</p>  <p>(auto-simplify setting off) (auto-simplify setting on)</p>	

Figure 3. Extract from Graspable's "Frequently Used Gestures" tutorial ([Graspable Team, 2020](#)). The symbols shake when students attempt an incorrect operation.

Reinventing the teaching and learning of proof with Graspable Math

Goal State

Make the expression match the goal!

$$k(3k+2) + 6(k+1) - 1$$

Steps: 0

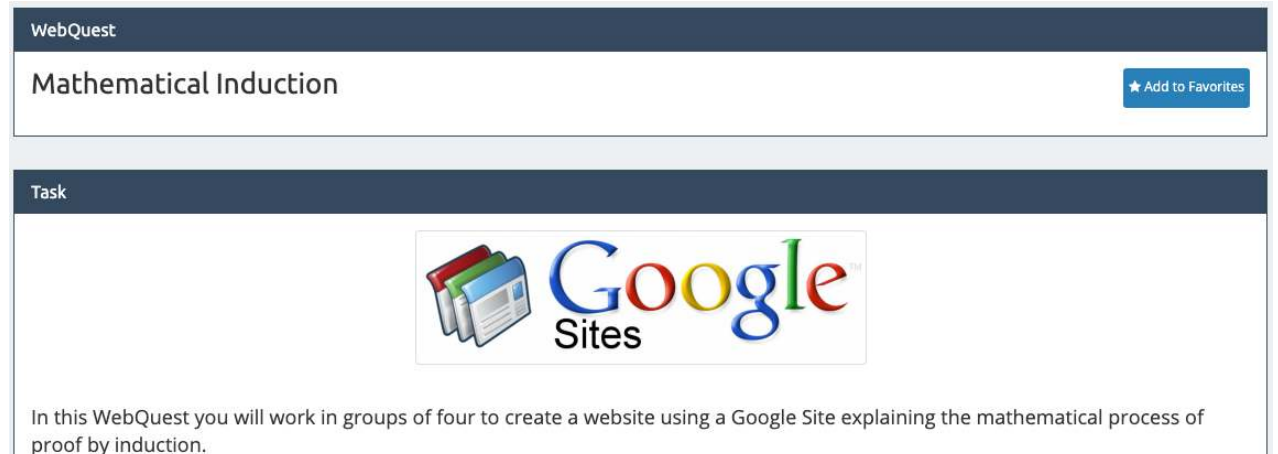
Goal: $(k+1)(3(k+1)+2)$

UNDO RESET KEYPAD

Figure 4. Proof by induction example suitable for Grade 10 students, created by the author.

Explanation

- Ideally, the explanation should be student-generated ([BSCS Science Learning, 2020](#))
- For weaker students, this phase could be modified to include a structured inquiry worksheet or a cloze activity



The screenshot shows a web page with a dark blue header bar containing the text "WebQuest". Below this, the title "Mathematical Induction" is displayed in a white box, with a blue "Add to Favorites" button on the right. A second dark blue header bar contains the word "Task". The main content area features a logo for Google Sites, which includes a stack of colorful folders and the word "Google" in its signature font, with "Sites" written below it. Below the logo, a paragraph of text reads: "In this WebQuest you will work in groups of four to create a website using a Google Site explaining the mathematical process of proof by induction."

Figure 5. Proof by Induction “WebQuest” suitable for older students ([Wathall, n.d.](#)).

Using Flipgrid to develop student voice

The screenshot shows a video player interface for a Flipgrid post. The video content includes a tree diagram for two coin flips. The first flip branches into 'Head' (0.9) and 'Tail' (0.1). The 'Head' branch further branches into 'Head' (0.9) and 'Tail' (0.1). The 'Tail' branch further branches into 'Head' (0.9) and 'Tail' (0.1). The final outcomes are HT, TH, and TT. Handwritten blue text on the video explains the probabilities for various outcomes.

Abhay Rao
Explanation of Tree Diagram - Math
Feb 14, 2020 5:41pm 15 views

The probability of a biased coin landing heads up is 0.9
It is tossed twice.
Complete this tree diagram and hence answer the following:

Head 0.9
Tail 0.1
Head 0.9
Tail 0.1

Head HT
Tail TH
Head TH
Tail TT

a) What is the probability of getting Tails twice?
 $P(TT)$

b) What is the probability of not getting Tails twice?
 $P(\text{not } TT)$

c) What is the probability of getting the same result twice?
 $P(\text{same}) = P(HH) + P(TT) =$

d) What is the probability of getting Heads exactly once?

4) $P(\text{not same}) = P(HT) + P(TH) =$

0:20 | 4:59

Figure 6. Example ISHCMC student video explanation posted on Flipgrid ([Rao, 2020](#)).

Elaboration and Evaluation – business as usual

- The usual extension and enrichment activities and problems can be given to students as they are ready for them in the Elaboration phase
- Evaluation is both a summative assessment and a student reflection



Comparison with traditional methods

- Similar timeframes possible – students may need longer to explore before they are ready to explain, but by applying technology to the learning of algebra we can augment, modify and redefine certain tasks (as suggested in [Puentedura's 2013 SAMR model](#)) and thus gain that time back in other ways
- Students take a more active responsibility for their learning of mathematics and arrive at new mathematical understandings from prior knowledge, rather than waiting for the teacher to “show them”
- Teaching role rebalances towards facilitation of discovery rather than direct instruction

Conclusion

The main challenge in applying the 5E model to the teaching and learning of high school algebra is finding ways to implement the “Engage” and “Explore” phases before the “Explain” phase

A variety of technology tools that have the potential to redefine the teaching and learning of high school algebra are now available

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